Name:

Fall 2015 Math 245 Exam 1

Please read the following directions:
Please write legibly, with plenty of white space. Please print your name on the designated line, similarly to your quizzes. Please fit your answers in the designated areas. To get credit, you must also show adequate work to justify your answers. If unsure, show the work. All problems are worth 5-10 points. The use of notes, calculators, or other materials on this exam is strictly prohibited. This exam will last at most 75 minutes; pace yourself accordingly. Please leave **only** at one of the designated times: 10am, 10:15am, 10:30am, 10:45am. At all other times please stay in your seat, to ensure a quiet test environment for others. Good luck!

Problem	Min Score	Your Score	Max Score
1.	5		10
2.	5		10
3.	5		10
4.	5		10
5.	5		10
6.	5		10
7.	5		10
8.	5		10
9.	5		10
10.	5		10
Total:	50		100

Problem 1. Carefully define each of the following terms:

a. contrapositive

b. valid

c. tautology

d. vacuous proof

e. proof by contradiction

Problem 2. Define the terms “proposition” and “predicate”, and explain the difference.

Problem 3. Write the negation of the proposition $\forall x \in \mathbb{R}, \exists y \in \mathbb{Z}, \forall z \in \mathbb{R},\ x > zy$, and simplify your result to eliminate $\sim$.

Problem 4. Construct the circuit corresponding to the Boolean expression $(p \wedge q) \vee \sim r$.

Problem 5. Write the converse of the inverse of the contrapositive of $p \to (q \vee r)$.

Problem 6. Use a truth table to determine whether $(p \oplus q) \vee r \equiv p \oplus (q \vee r)$.

Problem 7. Disprove the following statement: $\forall x \in \mathbb{R}$, if $x > 0$ then $\frac{1}{x+2} = \frac{1}{x} + \frac{1}{2}$.

Problem 8. Fill in the missing justifications, including line numbers, for the following proof.

1.	$(p \vee q) \to r$	hypothesis
2.	$\sim q \to c$	hypothesis
3.	p	hypothesis
4.	q	
5.	$p \vee q$	
6.	r	
7.	$\therefore p \wedge r$	

Problem 9. Carefully state the definition of $\lceil x \rceil$, and find some $y \in \mathbb{R}$ with $\lceil y \rceil > y^2$.

Problem 10. Use mathematical induction to prove that, for all natural $n \geq 2$,

$$2 + 3 + \cdots + n = \frac{(n-1)(n+2)}{2}.$$